Scaling exponents and transition behavior of the generalized conserved lattice gas model at zero temperature

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We study a generalized conserved lattice gas model in two dimensions by introducing an effective temperature to the conserved lattice gas model, where the number of particles is conserved during the dynamical process. We apply Monte Carlo simulation with the Metropolis transition rate. At zero temperature we find two transition behaviors; one between the localized active states and absorbing states and the other between the localized active states and active states. With a different definition of the order parameter for the second transition behavior, we obtain the critical exponents at the transition point.

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I. INTRODUCTION

Monte Carlo simulation in statistical models plays a major role in studying phase transitions and critical phenomena. Monte Carlo simulations in nonequilibrium have been performed in various diffusion processes [1–11]. The critical behavior of nonequilibrium phase transition is similar to that of equilibrium phase transitions in many respects. Therefore the critical behavior of nonequilibrium phase transitions can be understood well by the concept of scale invariance introduced to understand equilibrium phase transition.

Nonequilibrium phase transition from a fluctuating phase to one or several absorbing states of the conserved lattice gas (CLG) model was well studied [12–17], and it exhibits an absorbing phase transition from an active phase into an inactive phase. The CLG model was introduced by Rossi and co-authors [12]. In this model, there is no particle creation and annihilation process, that is, the number of particles is conserved during the dynamical process. At each time step, a randomly selected particle hops to a vacant nearest-neighbor site, provided that one of its nearest-neighbor sites has already been occupied. If an active particle cannot find a vacant nearest-neighbor site, the particle is immobile. A particle is called active if at least one of its neighboring sites is occupied by a particle. Applying this rule mentioned above to the CLG model in one dimension, many absorbing states below the critical particle density ρ_c, two symmetric absorbing states at ρ_c (checkerboardlike pattern), and an active state above ρ_c are observed [18]. For the CLG model in two dimensions, many absorbing states are observed both below and at $\rho_c=0.344\,94(3)$ [13], and an active state is observed above ρ_c. However, there are no two symmetric absorbing states in the CLG model in two dimensions. And then, Lübeck [13] has obtained the upper critical dimension of the CLG model and the critical exponents in two, three, four, and five dimensions. He has also studied persistence exponents [15] and the critical behavior on random neighbor space [16].

For further study of the CLG model, we introduce an *effective temperature* to the CLG model, and we call the CLG model with effective temperature the generalized CLG (GCLG) model. Here, the effective temperature plays a role of a noise parameter. In the CLG model, particles can move anywhere if they have an occupied neighborhood. When the effective temperature is introduced to the CLG model, particles are limited to move and they can move to the vicinity site when they reduce the number of neighborhoods. Though the terminology of *temperature* is not suitable to use for a nonequilibrium model, we use it in this paper to use the Metropolis-like update scheme. Using the effective temperature, we come to know that the GCLG model in two dimensions at zero temperature always minimizes the total number of bonds during the relaxation process. The dynamics in the GCLG model at zero temperature is exactly the same as that in the CLG model in one dimension. However, we observe the distinctive localized active phase in the GCLG model in two dimensions at zero temperature instead of two symmetric absorbing states shown in the CLG model in one dimension at the critical point, and also observe the absorbing state below ρ_c of the CLG model in two dimensions.

In this paper we consider the finite-size scaling as well as the dynamical scaling behavior of the GCLG model in two dimensions at zero temperature. First, we introduce a method that allows us to study our model at zero temperature. In addition to the previous attempt to measure finite-size effects in the absorbing phase in the CLG model, we use the order parameter to describe the localized active states in the GCLG model at zero temperature. We also apply the dynamical transition probability with the effective temperature to our model using the Metropolis rate.

II. THE GCLG MODEL

The *configuration energy* [7,8] of the GCLG model on an $L\times L$ square lattice in two dimensions, with fully periodic boundary conditions, can be specified as follows:

$$\mathcal{E}[\sigma]=J\sum_{\langle i,j\rangle}\sigma_i\sigma_j, \tag{1}$$

where the sum runs over the nearest-neighbor bonds of sites, and $J>0$ denotes the repulsive binding energy and each site

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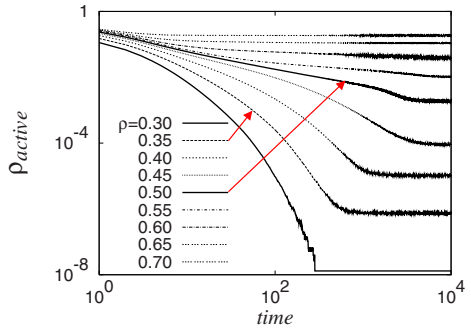

FIG. 1. (Color online) Plot of the density of active particles on $L=160$.

i is either occupied by a particle or empty, which we denote by a variable σ_i taking two values: 1 (occupied) or 0 (empty). In order to access the absorbing critical point, we change the initial particle density: $P_i=\Sigma_i\sigma_i/L^2$. Actually, Eq. (1) represents the number of bonds which are two neighboring occupied sites. In this paper, however, we will use the configuration energy instead of the number of bonds, though the GCLG model is not an equilibrium model.

In our simulation, the Metropolis rate [7,8,19,20] is implemented to the GCLG model in two dimensions. For the Metropolis rate, first we choose an active particle σ_i in the active list. Then, we randomly select a nearest-neighbor bond i, j of sites with different occupancies, $\sigma_i \neq \sigma_j$. We denote the original configuration as σ and let σ' be the configuration with σ_i, σ_j switched (i.e., $\sigma_i'=\sigma_j$; $\sigma_j'=\sigma_i$). The transition from σ to σ' is controlled by the Metropolis rate function as follows:

$$w(x) = \min(1, e^{-x}). \tag{2}$$

To be specific, we first compute $x=\beta\Delta$, where Δ denotes the configuration energy difference of two configurations (σ' and σ), and $\beta=1/k_BT$. If $x\leqslant 0$, the attempt (exchange) is accepted; if, however, $x>0$, we draw a random number rng and perform the exchange only if $rng\leqslant e^{-x}$, where random numbers are selected uniformly from the interval [0,1). As effective temperature increases, this fraction approaches 1 in a monotonic fashion. If this fraction is 1 at infinite temperature, the dynamic rule in the GCLG model in two dimensions with Metropolis rate is identical to that of the CLG model in two dimensions.

III. RESULTS

We have simulated the GCLG model at zero temperature using the Metropolis rate with various system sizes and initial particle density ρ.

A. Characteristics of dynamics

We have observed the density of active particle ρ_{active} in Fig. 1. In the CLG model, the critical particle density is 0.344 94(3) [13]. However, it seems that the GCLG model at $T=0$ has a different critical particle density comparing with that of the CLG model from Fig. 1. At the critical particle density of the CLG model, ρ_{active} decays exponentially while it decays with the power law over time in the CLG model. In this model, at $\rho=0.50$, ρ_{active} is decaying with the power law. Therefore we observed the two transition points $\rho_1=0.345$ and $\rho_2=0.50$ to look for the characteristics of dynamics.

Figure 2 is the plots of the density of active particles at the steady states ρ_s as a function of L. For $\rho<0.345$, ρ_s decays with a power law following the formula $\rho_s\sim L^{-\lambda}$ with $\lambda=2$, which means that active particles disappear as the lattice size goes to infinity. Therefore, for $\rho<0.345$, the GCLG at $T=0$ is in the absorbing state. For $\rho>0.345$, the plots are deviated from the guide line indicating $\rho_s\sim L^{-2}$. It exhibits that the GCLG is not in the absorbing state, because ρ_s is greater than $1/L^2$. Therefore we can find the first transition point ρ_1 between the active and absorbing states of the GCLG model at $T=0$.

Figure 3 is the plot of the density of active particles at $\rho=0.50$ as a function of time. Differently from the CLG model, the power-law relation between ρ_{active} and time appears at $\rho=0.50$, not around $\rho=0.345$. indicates the second transition point at $\rho_2=0.50$.

B. Characteristics of relaxation

What is characterizing the second transition, then? To find it out, we have assigned colors to particles (see Fig. 4) and have studied the GCLG model using colored particles. The particles are assigned to black, red, yellow, or blue. There are

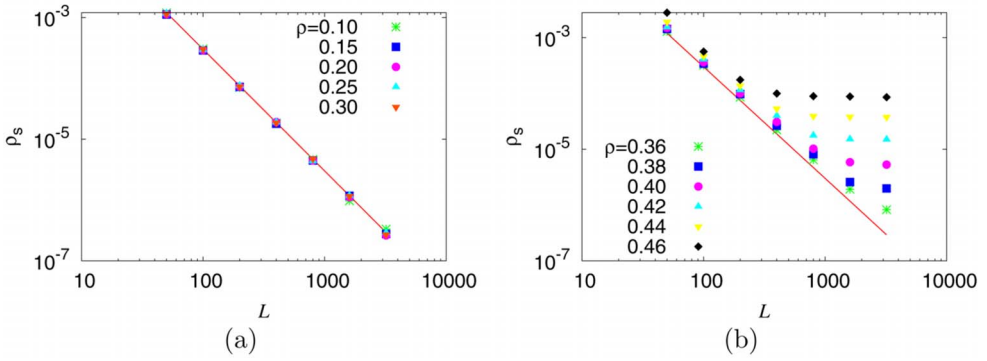

FIG. 2. (Color online) Plots of the density of active particles at the steady state ρ_s as a function of L for (a) $\rho<0.345$ and (b) $\rho>0.345$. The solid line is the guide line indicating $\rho_s\sim L^{-2}$.

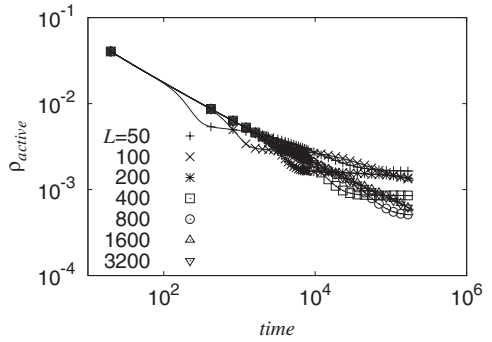

FIG. 3. Plot of the density of active particles on $L=50$, 100, 200, 400, 800, 1600, and 3200 at $\rho=0.50$ as a function of time for the GCLG model at $T=0$.

two kinds of black particles: one is a four-bonded particle and the other is a zero-bonded or isolated particle. To investigate the behavior of the GCLG model, we visualize our simulation with coloring of particles as follows: When an active particle performs a trial particle-hole exchange to all the possible directions, and if at least one of the particle-hole exchanges can reduce the configuration energy and the number of bonds in the system after a particle-hole exchange, it is colored red. If at least one of the particle-hole exchanges can maintain them, it is colored yellow. If a particle-hole exchange increases them, it is colored blue. If a particle is inactive because it is surrounded by particles or isolated, it is colored black. In Fig. 4, a randomly chosen active particle (empty square) could increase or decrease the configuration energy and the number of bonds in a system.

For a red particle, the number of bonds in the system are changed as follows: If a randomly chosen particle (empty square box) jumps to an empty site labeled by 1, the change in the number of bonds in system ΔN_b is reduced. If it jumps to an empty site labeled by 2, ΔN_b are increased. If it jumps to an empty site labeled by 3, ΔN_b are not changed. So, the

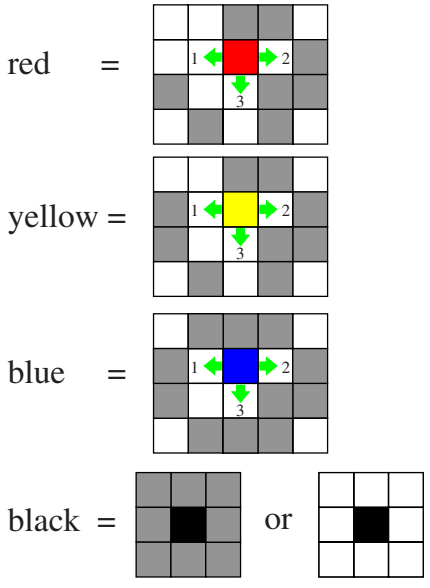

FIG. 4. (Color online) Schematic figures of coloring particles, where a randomly selected particle is indicated by an empty box.

minimum of ΔN_b for a red particle is $\Delta N_b^{\min}<0$. The minimum of ΔN_b for a yellow particle is $\Delta N_b^{\min}=0$, and for a blue particle, $\Delta N_b>0$.

At zero temperature, the blue particle is not an active particle because the increase of configurational energy is not allowed by the zero temperature dynamic rule, while it belongs to an active particle in the CLG model. The definition of active particles for the GCLG model at $T=0$ may consist of red and yellow particles, because these are only active particles at zero temperature.

For the CLG model in a steady state, there are two kinds of phases: one is an active phase and the other is many absorbing states. The absorbing states and the active states can be distinguished by the number of bonds: the bonds between occupied sites disappear at absorbing states, but the number of bonds and the configuration energy are fluctuating at active states. However, for the GCLG model at $T=0$ in a steady state, the remarkable characteristic is that the number of bonds and the configuration energy always become constant for any initial particle density.

In order to find out the second transition point ρ_2 precisely, we plot ρ_{red} as a function of time for different densities in Fig. 5 and we obtain $\rho_2=0.5000(25)$. For ρ_2, ρ_{red} is most stretched and it shows power-law decay behavior. Above second transition point ρ_2, any particle can visit any site in the system, whereas below ρ_2 and at ρ_2 the particles surrounded by inactive particles can be movable in the restricted region: we call this state the localized active state because it is active, but it is trapped and immovable.

We do not observe the power-law decay of active density which is constructed with red and yellow particles at ρ_1. The dynamical relaxation process of active particle density, which is constructed with both red and yellow particles together, is very complex, because the yellow particles in localized active states are never annihilated in the GCLG model at zero temperature. Thus we define the order parameter using the red particle density for studying the phase transition from the constant configuration energy active state to the localized active state. This order parameter describes well the phase transition in the GCLG model at zero temperature.

In the previous work, for the CLG model in two dimensions, the phase is separated by active and inactive (absorbing) phases with respect to the critical point. However, the GCLG model at $T=0$ is characterized by the absorbing phase mixed with the localized active state. The localized active state is a phenomenon in the GCLG model at $T=0$. To describe the complex localized active state, we devise a different order parameter.

Figure 6 shows the typical configurations of a localized active state in the system at zero temperature over time. Figure 6(a) shows the initial configuration of the GCLG at $T=0$ with linear dimension $L=100$ and initial density of particle $\rho=0.5$. In Figs. 6(b)–6(f), we can observe that the size of domain boundaries connected by the yellow particle goes on being reduced to minimize the configuration energy in the system. During this relaxation process the yellow particles located around boundaries create and annihilate the other particles with different colors. Figures 6(g) and 6(h) show that the boundaries become disconnected and the red par-

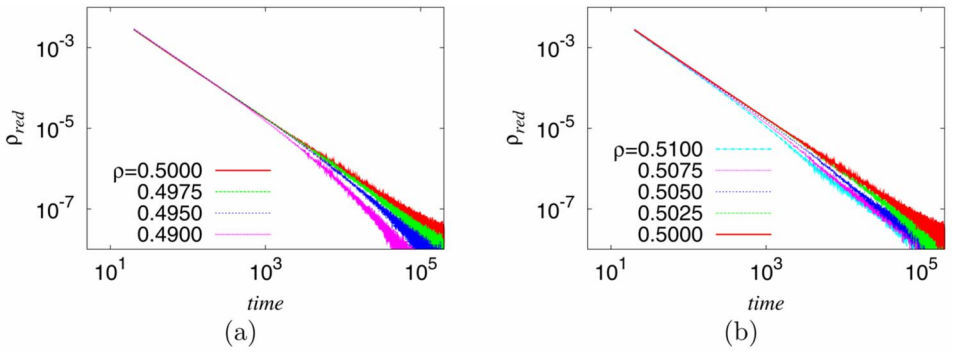

FIG. 5. (Color online) Plots of the density of red particles as a function of *time* for various particle densities: (a) is for $\rho \leqslant 0.5000$ and (b) is for $\rho \geqslant 0.5000$.

ticles disappear completely. In this steady state, we can observe that the red particle disappears and that the yellow particle oscillates in the restricted region.

Near initial density $\rho=0.5$, the typical configuration of the GCLG model at $T=0$ shows that the particles surrounded by inactive particles are trapped in the restricted region forever.

For the GCLG model at $T=0$, the red particles disappear in the steady state. At the second transition point ρ_2, we observe the power-law decay of the red particle density ρ_{red}. Comparing with Fig. 3, which is the plot of ρ_{active}, the power-law decay trend is clearer in Fig. 7 than the plot of ρ_{active} due to eliminating localized active, yellow particles. After reaching the steady state (yellow) particles do not actually play a role as active particles, because they are trapped in the restricted area. Therefore it is reasonable that ρ_{red} is used as the order parameter at $T=0$.

At the thermodynamic limit and the critical point for nonequilibrium continuous phase transition, the critical exponents are written as $\rho_{order} \sim (\rho-\rho_c)^{\beta}$, $\xi_{\perp} \sim (\rho-\rho_c)^{-\nu_{\perp}}$, and $\tau \sim (\rho-\rho_c)^{-\nu_{\parallel}}$, where ρ_{order} is the order parameter, $\xi_{\perp}$ is the correlation length, τ is the correlation time, and ρ is the particle density [5,6]. From these relations, we get the finite size scaling forms; $\rho_{order} \sim t^{-\delta} f(tL^{-z})$ and $\rho_{order} \sim L^{-\beta/\nu_{\perp}} g(tL^{-z})$, where the decay exponent is $\delta=\beta/\nu_{\parallel}$, the dynamic exponent is $z=\nu_{\parallel}/\nu_{\perp}$, and f and g are universal scaling functions.

Figure 7 shows that δ using the density of red particles is estimated by 1.282(1). During the processing of minimizing energy along the domain boundary, the red particle density is fluctuating. Figure 8 shows $\beta/\nu_{\perp}=2.00(5)$ and $z=1.55(5)$, where $\beta/\nu_{\perp}$ is known as the typical exponent of absorbing systems.

IV. CONCLUSIONS

We have introduced the GCLG model and simulated it on a two-dimensional lattice at zero temperature using the Metropolis transition rate. We find that the CLG model introduced by Lübeck [13] identifies with the GCLG model at infinite temperature because particles in the CLG model can move to any direction unless they are in the absorbing state.

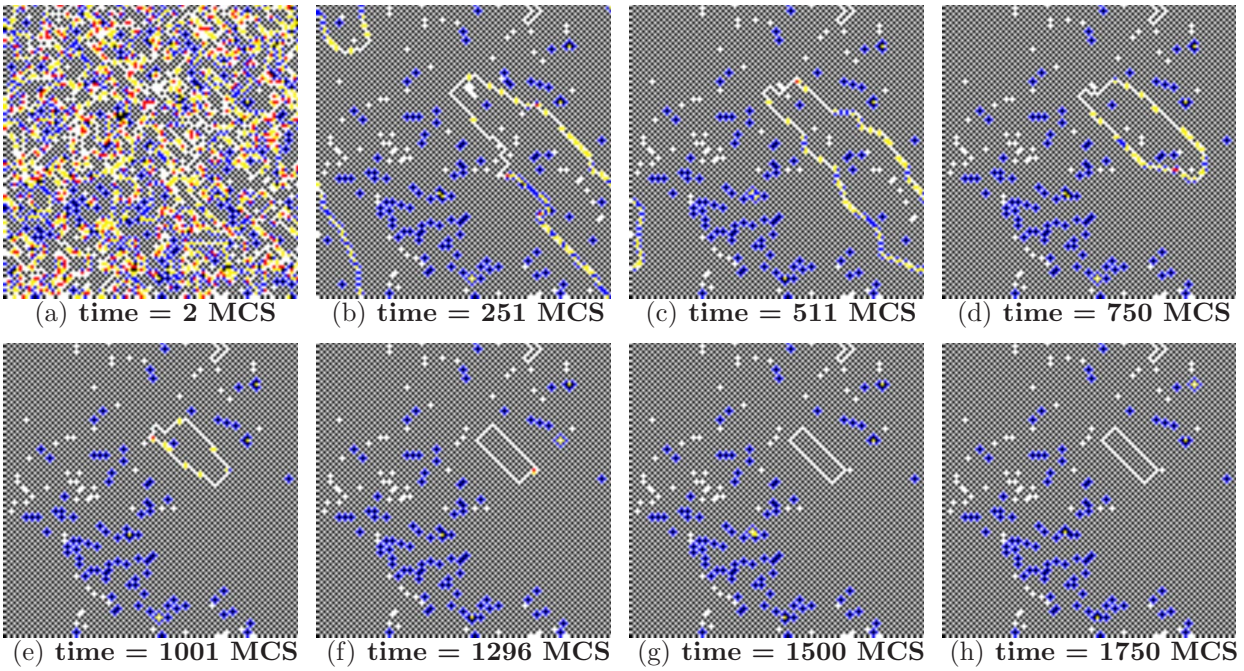

FIG. 6. (Color online) Schematic pictures of the localized active state for the GCLG model at zero temperature and initial density $\rho=0.50$ on a square system with linear dimension $L=100$.

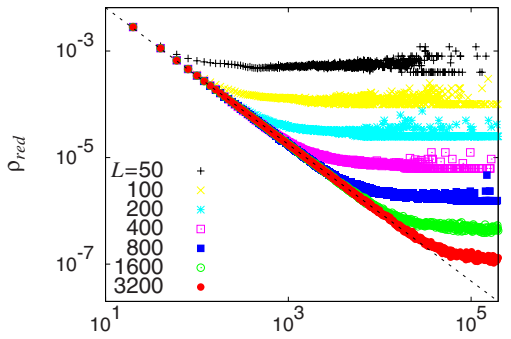

FIG. 7. (Color online) Plots of the density of red particles as a function of *time* on $L=50$, 100, 200, 400, 800, 1600, and 3200 at ρ_2, where $\delta=1.282(1)$.

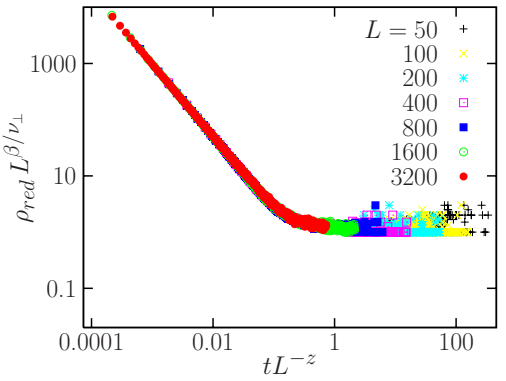

FIG. 8. (Color online) Plot of surviving sample density of red particles times $L^{\beta/\nu_\perp}$ as a function of tL^{-z} on $L=50$, 100, 200, 400, 800, 1600, and 3200 at ρ_2, where $\beta/\nu_\perp=2.00(5)$ and $z=1.55(5)$.

We observe two transition points, $\rho_1=0.345$ and $\rho_2=0.5000(25)$, for the CGLG model at zero temperature. The first transition point ρ_1 between the localized active states and the absorbing states is identical to the critical point ρ_c of the CLG model, but we do not observe power-law behavior at ρ_1. At the second transition point ρ_2 between the localized active states and the active states, we observe the power-law behavior of ρ_{active}, but we cannot apply the finite size scaling to the GCLG model by using ρ_{active} because it takes too long a time to measure the steady state value of ρ_{active} corresponding to the system size L.

The active particles of the GCLG model at zero temperature consist of the red particles and the yellow particles, where the yellow particles in localized active states are never annihilated. However, the red particles disappear at steady states below the second transition point ρ_2. Thus we define the red particle density as an order parameter for the GCLG model at $T=0$ to describe the second transition point ρ_2 between the localized active states and the active states. Then, we obtain the critical initial density ρ_2 in the GCLG model, and the critical exponents at ρ_2 are $\delta=1.282(1)$, $\beta/\nu_\perp=2.00(5)$, $z=1.55(5)$. Usually, the critical exponents for a nonequilibrium model are strongly related to the underlying dynamics. Thus the key dynamics and the critical exponents of a model are able to be guessed by mapping a model to a well-known model. Unfortunately, however, the GCLG model is not well fitted to the critical exponents as well as the underlying dynamics of well-known models. Therefore our further work is to find the transition behavior and the underlying dynamics of the GCLG model using the result at the nonextreme temperature, $0<T<\infty$, as well as $T=0$ and $T\to\infty$; nevertheless, our results imply that the coloring scheme of movable particles helps us to analyze and investigate the phase transition of a complex system such as the GCLG model at $T=0$.

ACKNOWLEDGMENTS

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